I: The first problem (4 points)

(a - 2 points) Even though we do not have the good prices as usual, the fact that the utility is a function of the expenditures means we do not need to know the prices. We have the following optimization problem:

$$maxU(x_1, x_2) \text{ subject to } x_1 + x_2 = 400 \tag{1}$$

An's optimal consumption bundle will be $x_1 = x_2 = 200$.

(b - 2 points) When the government subsides on food, the price of food that An has to pay decrease by 50%. Under this price system, An can spend up to \$400 on clothes or up to \$800 on food.

However, this price system ends when the subsidy amount hits \$100. That means, An can only spend up to \$200 on food on this subsidized system. Beyond this threshold, the price system reverts back to the old one without subsidy.

II: The second problem (4 points)

Binh's preferences over consumption bundles (x, y) are summarized by the following utility function:

$$U(x, y) = 16x - 2x^2 + 4y \tag{2}$$

where x is the amount of good x that Binh consumes and y is the amount of good y that Binh consumes. Let p_x and p_y be the prices of goods x and y respectively. Let m be Binh's income. Binh's goal is to maximize his utility subject to her budget constraint.

(a - 1 point) Binh's marginal rate of substitution between x and y can be given by:

$$MRS = \frac{U'_x}{U'_y} = 4 - x \tag{3}$$

.

The rate of substitution shows how many units of good y Binh is willing to exchange for one unit of good x.

(b - 1 point) Given Binh's budget contraint is $2x + 2y = 24$, we can replace $y = 12 - x$. Therefore we maximize the following function:

$$16x - 2x^2 + 4(12 - x) \tag{4}$$

As a result, Binh's optimal consumption bundle is $(3, 9)$.

(c - 1 point) With the price hike, the new budget constraint is $6x + 2y = 24$. Now we maximize the following function:

$$16x - 2x^2 + 4(12 - 3x) \tag{5}$$

As a result, Binh's optimal consumption bundle is $(1, 9)$. The utility is clearly down as Binh buys less of good x while buys the same amount of good y as before. This is because a price hike is a negative shock to the consumers like Binh.

(d - 1 point) Before the price hike, Binh's utility is 66. At the optimal consumption bundle we have:

$$MRS = \frac{p_x}{p_y} = 3 \tag{6}$$

which implies $x = 1$. To have the same utility as before, Binh must buy $y = 13$ which means his income must be $6x + 2y = 32$. Therefore to presever the same utility, he needs to receive an extra income of \$8.

We then have the following system of equations:

$$\begin{aligned} &16x - 2x^2 + 4y = 66 \\ &6x + 2y = m \\ &4 - 0.5x = 3. \end{aligned} \tag{7}$$

III: The third problem (8 points)

(a - 2 points) The profit function of each firm is

$$\Pi = P.q = (60 - Q' - q)q \tag{8}$$

where Q' is the total output of the other firms. Note that the production cost is 0. Given Q', the optimal response for firm 1 (and 2 and 3) is

$$q = \frac{60 - Q'}{2} \tag{9}$$

(b - 1 point) As the three firms are symmetric, the Cournot equilibrium is $q_1 = q_2 = q_3 = 15$.

(c - 3 points) When Firm 2 and Firm 3 merge, the best response of firm 1 is still as in Equation 9. The best response of the merged company is

$$Q* = \frac{60 - q_1}{2} \tag{10}$$

In this case, firm 1 will produce 20 and the total output of the merged company is also 20. Firm 2 and Firm 3 then each produces 10.

Now before the merger, the price was $60 - 45 = 15$. After the merger the price is $60 - 40 = 20$. Firm 1 is obviously better off as they can sell more units at a higher price. Firm 2 and Firm 3 are worse-off because their profit drops from $300 (=15*$20) to $200 (=10*$20). So it was a bad idea for Firm 2 and Firm 3 to merge.

(d - 2 points) In a cartel, the profit function is

$$\Pi = P.Q = (60 - Q)Q \tag{11}$$

which implies that the optimal output is $Q = 30$. In this case the total profit is $900 which is $300 each firm. This is higher than before when each firm receives $225.

IV: The fourth problem (4 points)

(a - 1 point) Suppose you have no data.

i) As we have no data, we cannot decide between these two bundles.

(b - 3 points) Suppose that you observe that when $p_x = 1, p_y = 1, m = 10$ the consumer chooses $x = 2, y = 8$. This shows that when $p_x = 1, p_y = 1, m = 5$, the consumer will choose $x = 1, y = 4$.

i) We have $(3, 6) \succeq (1, 4) \succeq (4, 1)$.

ii) We have $(3, 8) \succeq (2, 8) \succeq (6, 4)$.

iii) When $p_x = 1, p_y = 1, m = 10$ the consumer chooses $x = 2, y = 8$. Therefore when $p_x = 1, p_y = 1, m = 2.5$ the consumer chooses $x = 0.5, y = 2$. So we have $(5, 2) \succeq (0.5, 2) \succeq (0, 2.5)$.