I: The first problem (4 points)

An consumes two goods: food (measured in dollars) and clothes (also measured in dollars). Let x_1 be the **amount** that An spends on food in a given month and let x_2 be the **amount** that An spends on clothes in a given month. An's preferences over consumption bundles (x_1, x_2) are summarized by the utility function:

$$U(x_1, x_2) = x_1 x_2 \tag{1}$$

An's monthly income is \$400.

(a - 2 points) What is An's optimal consumption bundle? Illustrate your answer with a neat and clear diagram showing An's budget line and indifference curves. Label the points at which the budget line intersects the axes and identify the optimal bundle.

(b - 2 points) Suppose now that the government implements a subsidy program for food. Specifically, for each dollar that An spends on food, the government will give An \$0.50 in cash, with the restriction that the total amount of cash that An receives from the government cannot exceed \$100. In a neat and clear diagram, graph An's budget line. Label the points at which the budget line intersects the axes and determine the coordinates of the kink point.

II: The second problem (4 points)

Binh's preferences over consumption bundles (x, y) are summarized by the following utility function:

$$U(x, y) = 16x - 2x^2 + 4y \tag{2}$$

where x is the amount of good x that Binh consumes and y is the amount of good y that Binh consumes. Let p_x and p_y be the prices of goods x and y respectively. Let m be Binh's income. Binh's goal is to maximize his utility subject to her budget constraint.

(a - 1 point) Find an algebraic expression for Binh's marginal rate of substitution between x and y. In addition, give a coincise explanation of the meaning of the rate of substitution.

(b - 1 point) Suppose that $p_x = p_y = 2$ and $m = 24$. What is Binh's optimal consumption bundle.

(c - 1 point) Suppose now that p_x increases from 2 to 6, while p_y and m remain the same. What are Binh's optimal choices for x and y in this case. Is Binh better off or worse off than he was before the price hike? Explain your answer.

(d - 1 point) How much income must Binh be given so that his utility will be the same as before the price hike?

III: The third problem (8 points)

Consider an industry with 3 firms, each having marginal costs equal to 0. The inverse demand curve facing this industry is

$$P(Q) = 60 - Q \tag{3}$$

where $Q = q_1 + q_2 + q_3$ is total output.

(a - 2 points) If each firm behaves as a Cournot competitor, what is firm 1's best response function (i.e. optimal choice given other firms outputs)? what is firm 2's and firm 3's best response function (i.e. optimal choice given other firms outputs)?

(b - 1 point) Calculate the Cournot equilibrium.

(c - 3 points) Firm 2 and Firm 3 decided to merge and form a single firm with marginal costs still equal to 0. Calculate the new equilibrium. Is Firm 1 worse off or better off? Was it a good idea for firms 2 and 3 to merge?

(d - 2 points) Would it be a good idea for all three firms to organize a cartel?

IV: The fourth problem (4 points)

Assume there is a consumer with weakly monotonic, convex preferences and who is a utility maximizer. For each of the following pairs of bundles, specify if bundle 1 is $\succeq$, $\preceq$ or uncomparable to bundle 2. Your answer must have an explanation to have full credits.

(a - 1 point) Suppose you have no data.

i) Bundle 1: $(x, y) = (3, 3)$, Bundle 2: $(x, y) = (6, 2.5)$.

(b - 3 points) Suppose that you observe that when $p_x = 1, p_y = 1, m = 10$ the consumer chooses $x = 2, y = 8$.

i) Bundle 1: $(x, y) = (4, 1)$, Bundle 2: $(x, y) = (3, 6)$.

ii) Bundle 1: $(x, y) = (6, 4)$, Bundle 2: $(x, y) = (3, 8)$.

iii) Bundle 1: $(x, y) = (5, 2)$, Bundle 2: $(x, y) = (0, 2.5)$.