

Suppose we have the following AR(2) process

$$y_t = 0.5y_{t-1} + 0.5y_{t-2} + \epsilon_t \quad (1)$$

where the innovation  $\epsilon_t$  is the white noise:

$$\epsilon_t \sim N(0, 1) \quad (2)$$

---

**1: Conditional expectations**

---

(a) Compute  $E(y_t | y_{t-1}, y_{t-2}, \dots)$

(b) Compute  $E(y_t, y_{t-2}, y_{t-3}, \dots)$

---

**2: Linear forecast - Linear projection**

---

We consider the following linear forecast

$$x_t = 0.5y_{t-1} + 0.5y_{t-2} \quad (3)$$

(a) If  $\epsilon_t$  are identically and independently distributed, is  $x_t$  a linear projection?

(b) If  $\text{cov}(\epsilon_t, \epsilon_{t-1}) = 0.3$  for any  $t$ , is  $x_t$  a linear projection?

---

**3: Optimal forecast**

---

What is the optimal forecast of  $y_t$ ?