

Suppose we have the following MA(1) process

$$y_t = 0.3y_{t-1} + \epsilon_t \quad (1)$$

where the innovation ϵ_t is the white noise:

$$\epsilon_t \sim N(0, 1) \quad (2)$$

1: Autocovariance

- (a) Compute $E(y_t)$
- (b) Compute $Var(y_t)$
- (c) Compute $cov(y_t, y_{t-5})$
- (d) What is the autocovariance generating function?

2: Impulse responses

- (a) If ϵ_{10} increases by 1, how much does y_{15} increase?
- (b) If ϵ_t increases by 1 for all $t > 9$, how much does y_{15} increase?

3: Filtering

- (a) Suppose we want to analyse the average of two consecutive years. What is the filtered process that will help us?
- (b) What is its autocovariance generating function?

4: Invertibility

- (a) Let

$$y_t = 0.5y_{t-1} + \epsilon_t \quad (3)$$

Is this process invertible? If the answer is yes, can we write y_t as a function of the innovations?

- (b) Let

$$z_t = 1.5z_{t-1} + \epsilon_t \quad (4)$$

Is this process invertible? If the answer is yes, can we write z_t as a function of the innovations?