1: Stationarity

Let Y_t be an $MA(1)$ process:

$$Y_t = 3 + 0.3\epsilon_t + 0.7\epsilon_{t-1} \tag{1}$$

where ϵ_t is the Gaussian white noise.

(a) Check if Y_t is covariance stationary.

(b) Check if Y_t is strictly stationary.

(c) Check if Y_t is ergodic.

2: Lag operators

(a) Develop $(1 - 0.2L + 0.5L^2 - 0.3L^3)y_t$

(b) Develop $(1 - 0.3L)^{-1}y_t$

(c) Develop $[(1 - 0.3L)(1 - 0.2L)]^{-1}y_t$

(d) Develop $[1 - L + 0.25L^2]^{-1}y_t$

3: Moving average

Suppose we have the following $MA(1)$ process:

$$y_t = 0.3 + \epsilon_t + 0.2\epsilon_{t-1} \tag{2}$$

where the innovation ϵ_t is the white noise:

$$\epsilon_t \sim N(0, 1) \tag{3}$$

(a) What is our expectation about the value of y_t?

(b) What is the variance of y_t?

(c) What is $cov(y_1, y_{t-10})$?

(d) Is this process weakly stationary? strict stationary?

(e) Is this process ergodic?

(f) If e_10 increases by 1, how much does y_{15} increase?

(g) If e_t increases by 1 for all $t > 9$, how much does y_{15} increase?